The PAMO in Tunis: August 2004

1. Let n be a positive integer. Prove that there does not exist any integer m such that

$$3n^2 + 3n + 7 = m^3.$$

2. Is $4\sqrt{4 - 2\sqrt{3}} + \sqrt{97 - 56\sqrt{3}}$ an integer?

3. 268 numbers are written around a circle. The sum of 20 consecutive numbers is always 75. In position 17 is the number 3, in position 83 is the number 4 and in position 144 is the number 9. What number is in position 210?

4. Three real numbers satisfy the following two properties:

    (i) The square of their sum equals the sum of their squares.

    (ii) The product of the first two numbers equals the square of the third number.

    Find these numbers.

5. The digits 1, 3, 7 and 9 each occur at least once in the decimal notation of the integer n. Show that some permutation of the digits of n gives a number which is a multiple of 7.

6. A quadrilateral $ABCD$ is inscribed in a circle so that AB is a diameter. Suppose that AB and CD intersect at I, AD and BC at J, AC and BD at K and consider a point N on AB. Prove that IK is perpendicular to JN if and only if N is the midpoint of AB.