1 The PAMO in Alger, Algeria: August 2005

PAMO in Alger, Algeria 2005: Day 1
Time: 4.5 hours

1. For any positive real numbers a, b and c, prove:

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \geqslant \frac{2}{a+b}+\frac{2}{b+c}+\frac{2}{c+a} \geqslant \frac{9}{a+b+c}.$$

2. Let S be a set of integers with the property that any integer root of any non-zero polynomial with coefficients in S also belongs to S. If 0 and 1000 are elements of S, prove that -2 is also an element of S.

3. Let ABC be a triangle and let P be a point on one of the sides of ABC. Show how to construct a line passing through P that divides triangle ABC into two parts of equal area.

PAMO in Alger, Algeria 2005: Day 2
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4. Let $\lfloor x \rfloor$ be the greatest integer less than or equal to x and let $\{x\} = x - \lfloor x \rfloor$. Find all x satisfying $\lfloor x \rfloor . \{x\} = 2005x$.

5. Noah has to fit 8 species of animals into 4 cages of the Ark. He plans to put two species of animals in each cage. It turns out that, for each species of animal, there are at most 3 other species with which is cannot share a cage. Prove that there is a way to assign the animals to the cages such that each species shares a cage with a compatible species.

6. Let $f\colon \mathbb{Z} \to \mathbb{Z}$ be a function such that $f(ab) \geqslant f(a) + f(b)$ for all a, $b \in \mathbb{Z} \backslash \{0\}$. Show that $f(a^n) = nf(a)$ for all $a \in \mathbb{Z} \backslash \{0\}$ and all $n \in \natural$ if and only if $f(a^2) = 2f(a)$ for all $a \in \mathbb{Z} \backslash \{0\}$.