Problem 1:

Do there exist numbers $x_1, x_2, \ldots, x_{2009}$ from the set $\{-1, 1\}$, such that

$$x_1x_2 + x_2x_3 + x_3x_4 + \ldots + x_{2008}x_{2009} + x_{2009}x_1 = 999\,?$$

Problem 2:

Point P lies inside a triangle ABC. Let D, E and F be reflections of the point P in the lines BC, CA and AB, respectively. Prove that if the triangle DEF is equilateral, then the lines AD, BE and CF intersect in a common point.

Problem 3:

Let x be a real number with the following property: for each positive integer q, there exists an integer p, such that

$$\left|x - \frac{p}{q}\right| < \frac{1}{3q}\,.$$

Prove that x is an integer.

Problem 4:

Consider n children in a playground, where $n \geq 2$. Every child has a coloured hat, and every pair of children is joined by a coloured ribbon. For every child, the colour of each ribbon held is different, and also different from the colour of that child's hat. What is the minimum number of colours that needs to be used?

Problem 5:

Find all functions $f : \mathbb{N}_0 \to \mathbb{N}_0$ for which $f(0) = 0$ and

$$f(x^2 - y^2) = f(x)f(y) \qquad \text{for all } x, y \text{ with } x > y,$$

where $\mathbb{N}_0$ is the set $\{0, 1, 2, ...\}$.

Problem 6:

Points C, E, D and F lie on a circle with center O. Two chords CD and EF intersect at a point N. The tangents at C and D intersect at A, and the tangents at E and F intersect at B. Prove that $ON \perp AB$.