First Day : 26th May 2010

Duration : 4 h 30

Instructions

- *No calculating devices (computers, calculators, slide rules, etc), books, notes (printed or handwritten) are allowed in the examination room.*
- *Pens, pencils, rulers and compasses **only** may be used.*

Exercise 1

a) Show that it is possible to pair off the numbers 1, 2, 3, … , 10 so that the sums of each of the five pairs are five different prime numbers.
b) Is it possible to pair off the numbers 1, 2, 3, … , 20 so that the sums of each of the ten pairs are ten different prime numbers?

Exercise 2

How many ways are there to line up 19 girls (all of different heights) in a row so that no girl has a shorter girl both in front of and behind her?

Exercise 3

In an acute-angled triangle ABC, CF is an altitude, with F on AB, and BM is a median, with M on CA. Given that BM = CF and <MBC = <FCA, prove that triangle ABC is equilateral.

Second Day : 27th May 2010

Duration : 4 h 30

Instructions

- *No calculating devices (computers, calculators, slide rules, etc), books, notes (printed or handwritten) are allowed in the examination room.*
- *Pens, pencils, rulers and compasses **only** may be used.*

Exercise 4

Seven distinct points are marked on a circle of circumference c. Three of the points form an equilateral triangle and the other four form a square. Prove that at least one of the seven arcs into which the seven points divide the circle has length less than or equal to $\frac{c}{24}$.

Exercise 5

A sequence $a_0, a_1, a_2, \dots, a_n, \dots$ of positive integers is constructed as follows:
- if the last digit of a_n is less than or equal to 5 then this digit is deleted and a_{n+1} is the number consisting of the remaining digits. (If a_{n+1} contains no digits the process stops.)
- otherwise $a_{n+1} = 9a_n$

Can one choose a_0 so that an infinite sequence is obtained?

Exercise 6

Does there exist a function $f : Z \to Z$ such that $f(x + f(y)) = f(x) - y$ for all integers x and y ?