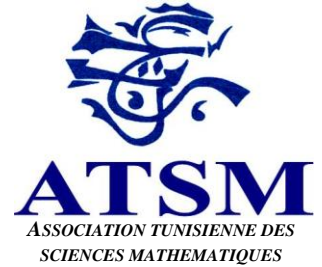




UNION MATHEMATIQUE AFRICAINE  
Commission Olympiades Pan  
Africaines de Mathématiques



21<sup>st</sup> edition of Panafrican Mathematics Olympiad  
Tunisia: 8 – 16 September, 2012

**First Day: 12<sup>th</sup> September, 2012**

**Duration : 4 h 30**

### Exercise 1

$AB$  is a chord (not a diameter) of a circle with centre  $O$ . Let  $T$  be a point on segment  $OB$ . The line through  $T$  perpendicular to  $OB$  meets  $AB$  at  $C$  and the circle at  $D$  and  $E$ . Denote by  $S$  the orthogonal projection of  $T$  onto  $AB$ .

Prove that  $AS \cdot BC = TE \cdot TD$ .

### Exercise 2

Find all positive integers  $m$  and  $n$  such that  $n^m - m$  divides  $m^2 + 2m$ .

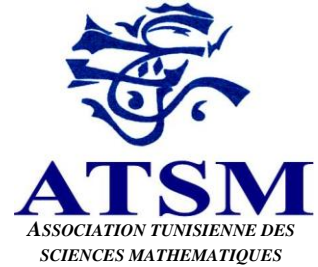
### Exercise 3

Find all real solutions  $x$  to the equation  $[x^2 - 2x] + 2[x] = [x]^2$ .

(Here  $[a]$  denotes the largest integer less than or equal to  $a$ . For example  $[7] = 7$ ,  $[7.3] = 7$  and  $[-4.2] = -5$ .)



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#### Exercise 4

The numbers  $\frac{1}{1}, \frac{1}{2}, \dots, \frac{1}{2012}$  are written on the blackboard. Aïcha chooses any two numbers from the blackboard, say  $x$  and  $y$ , erases them and she writes instead the number  $x + y + xy$ . She continues to do this until only one number is left on the board.

What are the possible values of the final number?

#### Exercise 5

Find all functions  $f: \mathbb{R} \rightarrow \mathbb{R}$  such that  $f(x^2 - y^2) = (x + y)(f(x) - f(y))$  for all real numbers  $x$  and  $y$ .

#### Exercise 6

(i) Find the angles of  $\triangle ABC$  if the length of the altitude through  $B$  is equal to the length of the median through  $C$  and the length of the altitude through  $C$  is equal to the length of the median through  $B$ .

(ii) Find all possible values of  $\angle ABC$  of  $\triangle ABC$  if the length of the altitude through  $A$  is equal to the length of the median through  $C$  and the length of the altitude through  $C$  is equal to the length of the median through  $B$ .