UNION MATHEMATIQUE AFRICAINE
Commission Olympiades Pan Africaines de Mathématiques

ATSM
ASSOCIATION TUNISIENNE DES SCIENCES MATHEMATIQUES

21st edition of Panafrican Mathematics Olympiad

Tunisia: 8 – 16 September, 2012

First Day: 12th September, 2012

Duration : 4 h 30

Exercise 1

AB is a chord (not a diameter) of a circle with centre O. Let T be a point on segment OB. The line through T perpendicular to OB meets AB at C and the circle at D and E. Denote by S the orthogonal projection of T onto AB.
Prove that $AS \cdot BC = TE \cdot TD$.

Exercise 2

Find all positive integers m and n such that $n^m - m$ divides m^2+2m.

Exercise 3

Find all real solutions x to the equation $[x^2 - 2x] + 2[x] = [x]^2$.
(Here $[a]$ denotes the largest integer less than or equal to a. For example $[7] = 7$, $[7.3] = 7$ and $[-4.2] = -5$.)

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Exercise 4

The numbers $\frac{1}{1}, \frac{1}{2}, \ldots, \frac{1}{2012}$ are written on the blackboard. Aïcha chooses any two numbers from the blackboard, say x and y, erases them and she writes instead the number $x + y + xy$. She continues to do this until only one number is left on the board.
What are the possible values of the final number?

Exercise 5

Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that $f(x^2 - y^2) = (x + y)(f(x) - f(y))$ for all real numbers x and y.

Exercise 6

(i) Find the angles of ΔABC if the length of the altitude through B is equal to the length of the median through C and the length of the altitude through C is equal to the length of the median through B.

(ii) Find all possible values of $\angle ABC$ of ΔABC if the length of the altitude through A is equal to the length of the median through C and the length of the altitude through C is equal to the length of the median through B.