

23<sup>rd</sup> edition of the Pan African Mathematics Olympiad

Abuja: 20 August - 29 August, 2015

First Day: 24 August 2015

Duration : 4 h 30

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1. Prove that

$$\sqrt{x-1} + \sqrt{2x+9} + \sqrt{19-3x} < 9$$

for all real  $x$  for which the left-hand side is well defined.

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2. A convex hexagon  $ABCDEF$  is such that

$$AB = BC \quad CD = DE \quad EF = FA$$

and

$$\angle ABC = 2\angle AEC \quad \angle CDE = 2\angle CAE \quad \angle EFA = 2\angle ACE.$$

Show that  $AD$ ,  $CF$  and  $EB$  are concurrent.

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3. Let  $a_1, a_2, \dots, a_{11}$  be integers. Prove that there are numbers  $b_1, b_2, \dots, b_{11}$ , each  $b_i$  equal  $-1, 0$  or  $1$ , but not all being  $0$ , such that the number

$$N = a_1b_1 + a_2b_2 + \dots + a_{11}b_{11}$$

is divisible by 2015.

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4. For a positive integer  $n$  denote  $d(n)$  its greatest odd divisor. Find the value of the sum  $d(1008) + d(1009) + \dots + d(2015)$ .

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5. There are seven cards in a hat, and on the card  $k$  there is a number  $2^{k-1}$ ,  $k = 1, 2, \dots, 7$ . Solarin picks the cards up at random from the hat, one card at a time, until the sum of the numbers on cards in his hand exceeds 124. What is the most probable sum he can get?

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6. Let  $ABCD$  be a quadrilateral (with non-perpendicular diagonals).

- The perpendicular from  $A$  to  $BC$  meets  $CD$  at  $K$ .
- The perpendicular from  $A$  to  $CD$  meets  $BC$  at  $L$ .
- The perpendicular from  $C$  to  $AB$  meets  $AD$  at  $M$ .
- The perpendicular from  $C$  to  $AD$  meets  $AB$  at  $N$ .

1. Prove that  $KL$  is parallel to  $MN$ .

2. Prove that  $KLMN$  is a parallelogram if  $ABCD$  is cyclic.

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