Problem 1: Let \( n \) be an integer greater than 3. A square of side length \( n \) is divided by lines parallel to each side into \( n^2 \) squares of length 1. Find the number of convex trapezoids which have vertices among the vertices of the \( n^2 \) squares of side length 1, have side lengths less than or equal to 3, and have area equal to 2. Note: parallelograms are trapezoids.

Problem 2: Let \( C \) be a circle, \( P \) be a point outside it, and \( A \) and \( B \) be the intersection points between \( C \) and the tangents from \( P \) onto \( C \). Let \( K \) be a point on the line \( AB \), distinct from \( A \) and \( B \) and let \( T \) be the second intersection point of \( C \) and the circle passing through \( P \), \( B \) and \( K \). Also, let \( P' \) be the reflection of \( P \) in point \( A \). Show that \( \angle PBT = \angle P'KA \).

Problem 3: Let \( a_0, a_1, \ldots \) and \( p_0, p_1, \ldots \) be infinite sequences of positive integers such that:

- \( a_0 \geq 2 \),
- \( p_n \) is the smallest prime divisor of \( a_n \) for each integer \( n \geq 0 \), and
- \( a_{n+1} = a_n + \frac{a_n}{p_n} \) for each integer \( n \geq 0 \).

Prove that there is an integer \( N \) such that \( a_{n+3} = 3a_n \) for \( n > N \).
Problem 4: Find all integers $m$ and $n$ such that

\[
\frac{m^2 + n}{n^2 - m} \quad \text{and} \quad \frac{n^2 + m}{m^2 - n}
\]

are both integers.

Problem 5: Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that for all $x, y \in \mathbb{R}$,

\[
(f(x) + y)(x + f(y)) = f(x^2) + f(y^2) + 2f(xy).
\]

Problem 6: Let $ABCD$ be a trapezoid, not a parallelogram, with $AD \parallel BC$. Let the diagonals $BD$ and $AC$ intersect at a point $O$. The circumscribed circles to triangles $AOB$ et $DOC$ meet again at the second point $S$.

Prove that the circumscribed circles to triangles $ASD$ and $BSC$ are tangent.